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Investigation of Three-Dimensional Turbulent Rectangular Jets

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The mean flowfield and turbulent intensities of airjets, issuing from retangular slots having different geometries and aspect ratios were measured using hot-wire anemometers. It is suggested that the anomalous behavior of the flow in the two-dimensional region of the jet is associated with three-dimensional effects. The nozzle geometry and aspect ratio play a very important role on the jet development through these three-dimensional effects. In comparison to orifice jets, flows out of rectangular channels are found to have a two-dimensional region which extends further downstream. Furthermore, the turbulent structure of these jets approach closer to a state of self-preservation.

Nomenclature

A = aspect ratio l/d
d,l = nozzle dimensions
J = momentum flux in the x,y plane:

$$\int_{-\infty}^{+\infty} [U(x,y,0)]^2 \mathrm{d}y$$

k = empirical constant of the velocity profile
K,L = centerline velocity decay constants
U = mean velocity in the x direction
U_c = mean velocity along the centerline (x axis)
U_j = space average of the mean velocity at $x=0$
u,v,w = fluctuating velocity components along x, y, and z
u',v',w' = turbulent intensities
$(uv)'$ = turbulent shear stress in the x,y plane: $\overline{uv}/U_c^2$
$\bar{x}$ = dimensionless x
x,y,z = rectangular coordinates
δ = distance from centerline to the point on the y axis where $U=U_c/2$
λ = distance from the centerline to the point on the z axis where $U=U_c/2$
η = dimensionless distance y/x
ζ = dimensionless distance z/x
θ = momentum thickness J/U_j^2

Introduction

TURBULENT jets find their applications in a variety of fields ranging from aeronautics to civil engineering. The need for a more detailed understanding of these flows has recently prompted a number of new investigations to be launched.

By far, most of the effort so far has been devoted to the study of the circular jet and to the two-dimensional jet. Jets issuing from rectangular nozzles, as shown in Fig. 1, are not necessarily two dimensional. The first investigations showing three-dimensional effects in such flows were carried out more than a decade ago.[1-3] More recently, velocity and temperature fields were measured.[4] Nozzles of rectangular and cruciform shapes have also been tested in conjunction with thrust augmentors.[5]

The flow out of a rectangular slot of thickness d and length l starts by developing into a two-dimensional jet as soon as the shear layers on the sides of the jet develop and reach the centerline. The two-dimensional region of the jet may or may not exist. Its existence and extent in the streamwise direction is strongly a function of the aspect ratio A and nozzle geometry. Further downstream, the jet tends toward an axisymmetric shape and resembles a jet issuing from a circular orifice.

The present work is a continuation of a previous study[6] which repeated some of the measurements of Sforza et al.[1] and led to a number of conflicting results. The lack of agreement centered mainly on the flow behavior in the so-called two-dimensional region of the jet where the centerline velocity decay is apparently a function of the aspect ratio A. In fact, a number of other discrepancies in the results of measurements of two-dimensional jets have long been known and found quite intriguing.

In an attempt to clear up these discrepancies, measurements of the mean flowfield and turbulent intensities in a number of jets were made using linearized hot-wire anemometers. However, most of the present work centers on a comparison between two jets issuing from nozzles with different geometries but having the same Re and A.

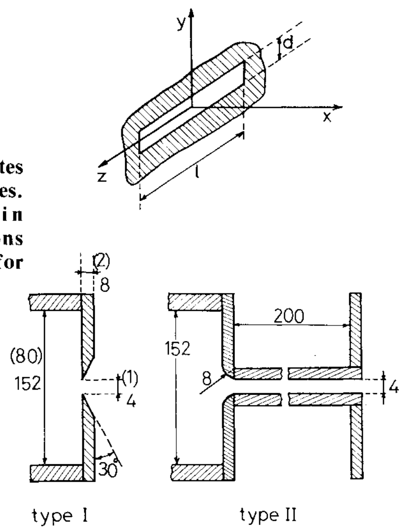

Fig. 1 Flow coordinates and schematics of nozzles. All dimensions in millimeters; dimensions between parentheses are for the small-size nozzles.

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Experimental Apparatus and Procedure

The experiments described herein were performed in the Mechanical Engineering Laboratories of the American University of Beirut. In all, ten nozzles were used, having dimensions as shown in Fig. 1. Nozzles referred to as type I consist of rectangular slots with sharp edges. Nozzles of type II consist of rectangular channels having a length of 20 cm.

The flow facility used with the smaller nozzles consisted of a cylindrical settling chamber on which the nozzle plates were fastened. Air was drawn from the laboratory compressed air line into this chamber via a filter, two pressure-regulators, a needle valve, and a fan-coil heat exchanger. The larger nozzles were mounted at the end of the contraction section of a small prototype wind tunnel described in Ref. 4.

Hot-wire and quartz-coated hot-film probes were used. Fluctuations in the y and z directions were measured using symmetric X-array probes. The probes were mounted on a traversing mechanism capable of motion along the three axes. The sensors were operated by Thermo-Systems Inc. constant-temperature linearized circuits. An analog correlator, four-quadrant multipliers, and rms converters were used to deduce u', v', w' and $\overline{uv}$. Turbulence levels measured previously [6] were 10-20% lower than the present ones, due to the inadequate low-frequency response of the rms circuits used earlier. In the present data, averaging times of the order of 100 s were used in the outer portions of the jet and in the far field. The mean flow measurements were corrected to account for the high turbulence levels. No corrections were applied to the fluctuating components as the high-order terms were not measured.

Experimental Results

Mean Velocity Field

Some features of the mean velocity profiles have been presented in Refs. 1 and 2 and more recently by Marsters.[5] Earlier measurements carried out on the same apparatus have been published elsewhere.[4,6] The present investigation having been interrupted for two years, some of the measurements made earlier were repeated. No noticeable changes in the mean flow data were observed.

Before introducing the new results, a summary of the older data is in order. Figures 2a-c show the variation of $(U_c/U_j)^2$ as a function of x/θ. Based on similar results, Sforza et al.[1] divide the flowfield in the x direction into three regions:

1) A jet flow establishment region where $\bar{U}_c = \text{const}$.

2) A characteristic decay region where $\bar{U}_c^n = K(\bar{x}-\bar{x}_0)$; the exponent n is generally a function of nozzle geometry. When $n=2$, this region becomes a two-dimensional flow region.

3) An axisymmetric region where $\bar{U}_c = L(\bar{x}-\bar{x}_0)$.

The dimensionless parameter $\bar{U}_c$ is obtained by dividing U_c by the jet velocity at exit or by the average velocity across the jet at exit. As for $\bar{x}$, it is obtained by dividing x by either the jet thickness d or by the momentum thickness of the jet. The latter parameter gives a better correlation and has been used in the present study.

This behavior is followed closely by the jet issuing out of nozzles of type II. Values of the constants K and L are found in Ref. 4. Data in the axisymmetric region can be made to collapse on a single curve if the nondimensional distance x is taken to be the ratio of x to the diameter of a circular nozzle, with the same momentum at exit as the rectangular nozzle.

The jets out of sharp-edge nozzles do not always have a region where $\bar{U}_c^2 \propto \bar{x}$. This is particularly true for theexperiments with the smaller nozzles. The flow out of the sharp-edged nozzle at $Re = 3700$ does not exhibit such a relationship when $A < 40$. Even at the higher aspect ratios, the slope of the centerline velocity decay deviates somewhat from the linear variation exhibited by the other jets. For the larger nozzles of the same type, a centerline decay representative of two-dimensional jets is found even for $A = 10$. The data for the smaller jets are quite similar to that reported by Sforza et al.[1]

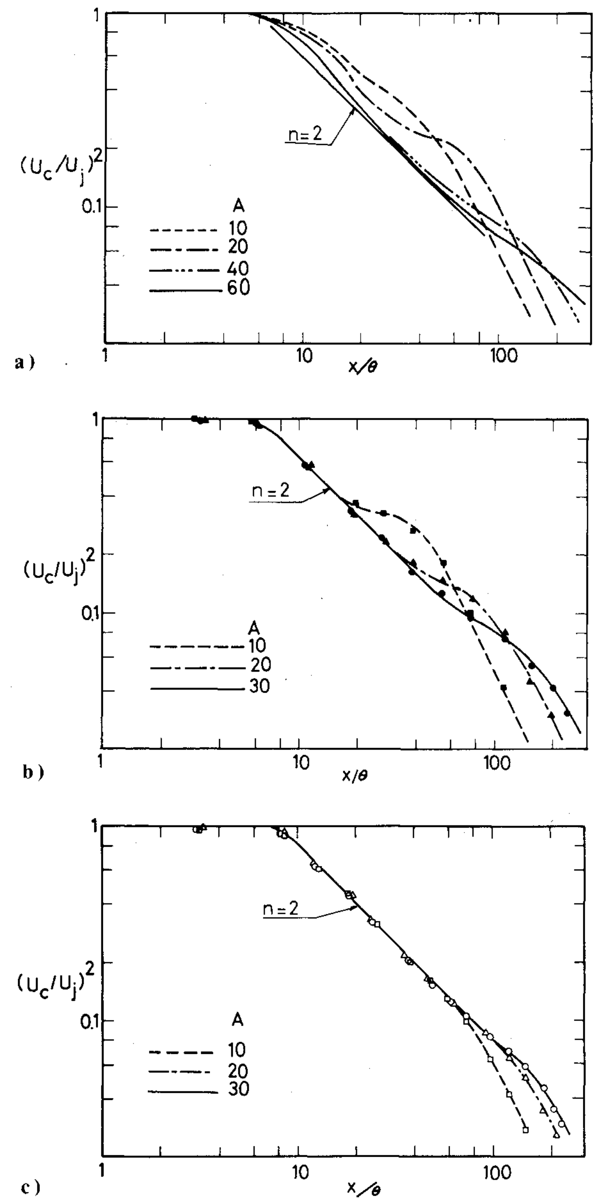


Fig. 2 Centerline velocity decay. a) $Re = 3700$, nozzle type I (small size); b) $Re = 12{,}200$, nozzle type I (large size); c) $Re = 12{,}200$, nozzle type II.

The jet spread in the two planes of symmetry x,y and x,z is shown in Fig. 3. The jet width δ is defined as the value of y for which $U = \frac{1}{2}U_c$, and λ is a similarly defined quantity along the z axis. These results show that, whereas the jet of nozzle type II spreads slightly in the x,z plane, the jet out of the sharp nozzle has the opposite behavior. In the region where $\delta < \lambda$, the variation of δ with x is linear to a first degree of approximation. The slopes are 0.115 and 0.104 for nozzles I and II, respectively. The first value is higher than most values reported in the literature. At a certain distance from the jet exit, δ outgrows λ; then, further downstream, the two parameters appear to be tending to have the same variation. At these large distances, the jet is presumably circular. This variation of δ and λ is more pronounced in the case of nozzle I where, furthermore, the length of the two-dimensional region is quite short in comparison with the other jet.

Detailed velocity profiles in the x,y plane are shown in Fig. 4. These profiles are found to be geometrically similar. A good empirical curve fit is given by $\bar{U} = \exp(-K\eta^2)$ where $K = 55$ for nozzles of type I and $K = 68$ for nozzles of type II.

Profiles in the x,z plane are shown on Figs. 5a and b. Both figures show an increase in U away from the center but,

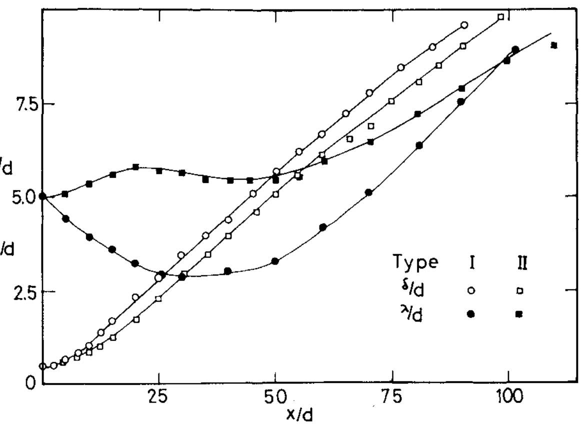


Fig. 3 Growth of the jet with downstream distance, $Re = 12{,}200$.

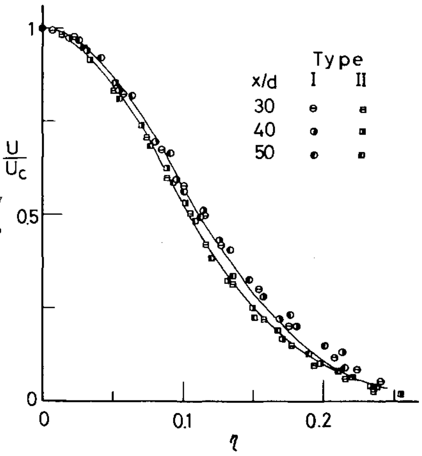


Fig. 4 Mean velocity profiles in the x,y plane, $Re = 12{,}200$.

whereas this increase is less than 5% in the case of nozzle II, it is between 10 and 20% for the other nozzle. These "saddle-back" shape profiles have been reported in a number of sources such as Refs. 1, 4 and 5. They are perfectly reproducible and are more noticeable for jets out of the smaller sharp-edged nozzles (not shown here). Conversely, they are barely detectable for jets out of nozzles of type II, particularly when A is large.

Here again, $\bar{U}(\zeta)$ tends toward similarity at large distances downstream.

Turbulence Measurements

The longitudinal turbulence level, $u' = u_{rms}/U_c^2$, on the centerline is shown in Figs. 6a-c. Variations in the turbulence intensity in the three regions of the jet are as follows:

1) In the region of jet flow establishment u' grows rapidly as soon as the shear layers, bounding the jet at the exit, reach the centerline. This increase is faster for nozzles of type I, although u' in such jets starts out from lower values.

2) In the so-called characteristic decay region of the jet, u' tends to a constant level of about 0.25. This is close to values reported recently in the literature.[7] However, this value is not reached in all jets tested. In the flow out of the sharp-edged nozzles, particularly the smaller ones, u' reaches a maximum well below the value of 0.25. Also, in these jets the turbulence level does not stabilize at a constant value for any appreciable length. It decreases rapidly before starting to increase again as we approach the axisymmetric-flow region.

3) In the axisymmetric region, and where measurements were taken at very large distances from the jet exit, u' seems to stabilize at a level close to 0.3. This is the same value reported by Wygnanski and Fiedler[8] in their measurements of a circular jet. For nozzle I when $A = 10$, u' seems to be approaching a higher value.

The dip which occurs in the region of transition from two-dimensional to axisymmetry is more pronounced for jets out of sharp nozzles. It is barely noticeable for the other jets.

The longitudinal velocity fluctuation profiles $u'(\eta)$ for $A = 10$ and for nozzles of both types I and II are shown in Figs. 7a and b. In this figure, the profiles are geometrically similar when $10 < (x/d) < 30$ for type I nozzle, and $30 < (x/d) < 50$ for type II nozzle. The geometric similarity of the turbulent intensity variation with y, when both quantities are nondimensionalized using the local velocity and length scales, usually indicates that the turbulence field has approached a state of local invariance or has become self-preserved. Strictly speaking, however, complete self-preservation is established when the turbulence field is in a state of dynamic equilibrium such that the local inputs of energy or turbulent energy production balances the local dissipation. In the present investigation, no measurements were taken of the higher-order turbulence parameters necessary to ascertain the existence of complete self-preservation. The assessment of whether the flow has approached self-preservation is based on the observation of turbulent intensities and stresses.

The data of Fig. 7b differ very slightly from the data of Heskestad[9] and that of Gutmark and Wygnanski,[7] and seem to indicate that the turbulent structure of that jet has approached a state of self-preservation 30 slot widths downstream of the jet exit. The other fluctuating velocity component profiles $v'(\eta)$ and $w'(\eta)$, shown in Figs. 8a and 8b, confirm this. These profiles closely resemble those of Heskestad.[9] Values of w' in the central part of the jet are about 5% higher than those reported in Ref. 7 while v' is 3% lower. On the centerline, the ratios $v'/u' = 0.74$ and $w'/u' = 0.83$. This shows that the turbulent structure is not isotropic, even in the central part of the jets where the production of turbulent energy is small. The variation of $u'(\eta)$ exhibits the usual "saddle-back" shape. The maximum value of $u'(\eta)$ occurs at $\eta = 0.75$ for both jets. However, the ratio (u'_{max}/u'_{center}) takes the values 1.21 and 1.10 for

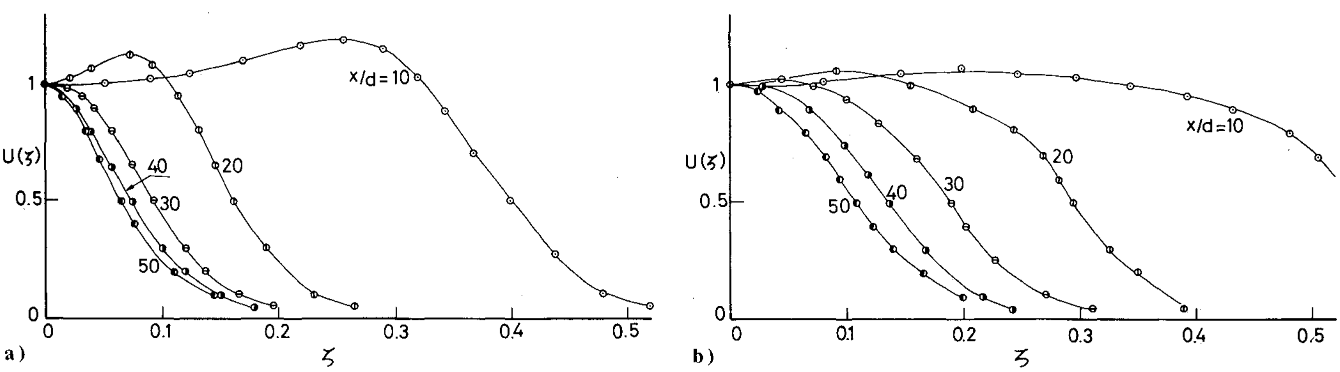


Fig. 5 Mean velocity profiles in the x,z plane: a) nozzle type I, $Re = 12{,}200$; b) nozzle type II, $Re = 12{,}200$.

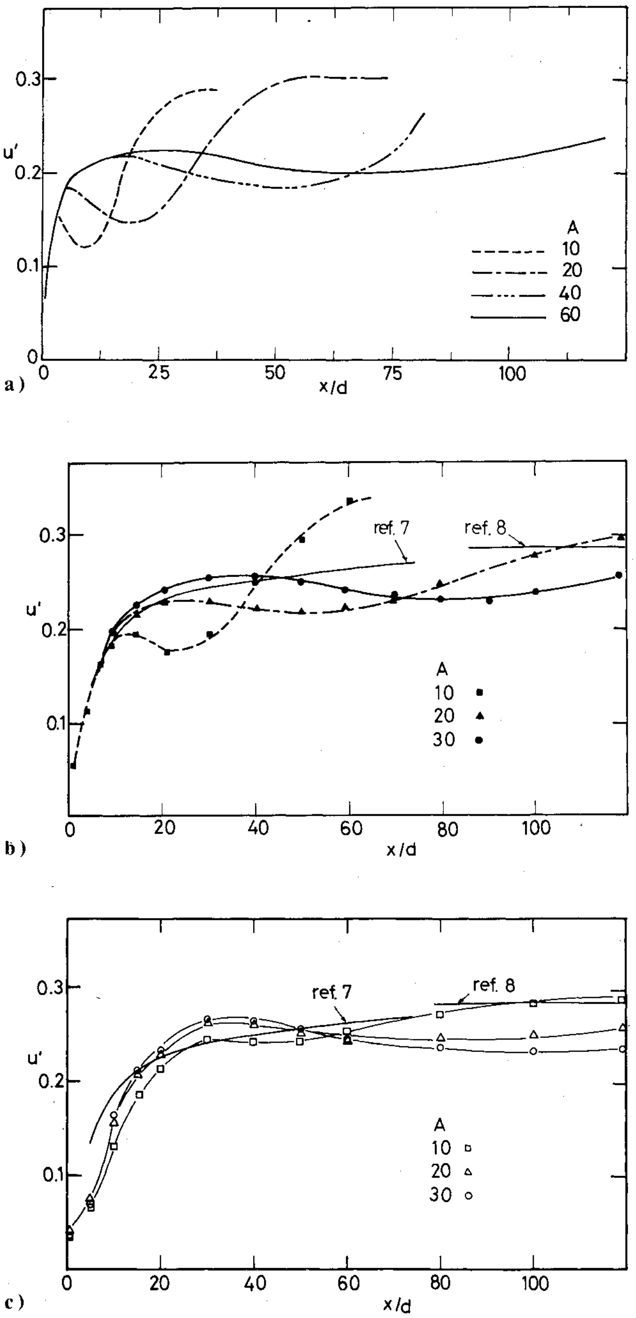


Fig. 6 Variation of the longitudinal turbulence intensity on the centerline with the downstream distance a) nozzle type I (small size), $Re = 3700$; b) nozzle type I, $Re = 12{,}200$; c) nozzle type II, $Re = 12{,}200$.

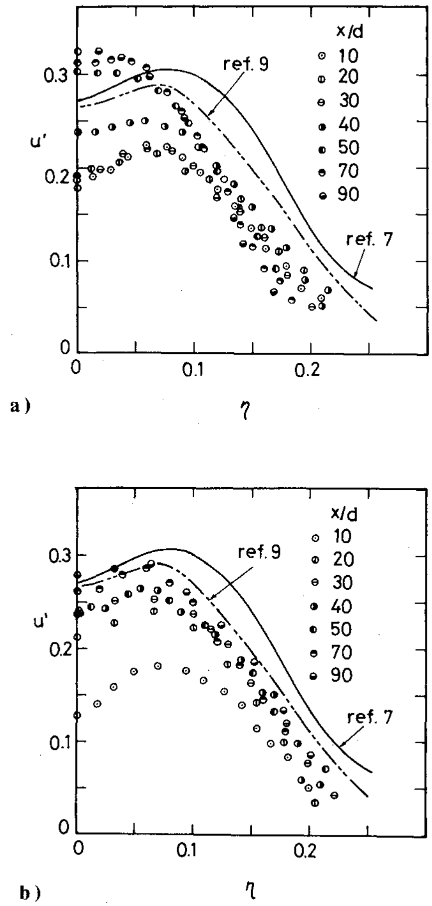


Fig. 7 Longitudinal turbulent intensity profiles in the x,y plane: a) nozzle type I, $Re = 12{,}200$; b) nozzle type II, $Re = 12{,}200$.

nozzles I and II, respectively. The value of this ratio is usually indicative of the state of self-preservation of a jet. Near the jet exit, in the region of jet flow establishment, this ratio is quite large, as all the turbulent energy production takes place in the two shear layers bounding the jet. A large value, therefore, indicates that the effects of the merging of the two shear layers bounding the potential core is still felt. The high value noted for the jet out of the sharp-edged nozzle suggests that, although the profiles $u'(\eta)$ are similar, the turbulent structure is still influenced by its past history and has not attained a state of local invariance. In the other jet, and due to the turbulence initially present in the central part of the jet at $x=0$, it does seem that the flow is closer to a self-preserved state.

The variation of the turbulent shear stress as a function of η is shown in Fig. 9. Good agreement is obtained between the calculated profile and measured values. The calculated profile was deduced from the relation:

$$(uv)'/U_c^2 = U/2U_c \int_0^{\eta} (U/U_c)\,d\eta$$

which is the integral of the momentum equation assuming similarity conditions. This equation gives large scatter in the values of $(uv)'$ if one uses measured velocity ratios U/U_c. In order to avoid this, the empirical curve fit $U/U_c = e^{-k\eta^2}$ was used instead.

The maximum value of the shear stress occurs at around $\eta = 0.72$, and the value of this maximum is about 2-3% lower than in Ref. 7. The broken line on the same figure shows the calculated distribution of turbulent shear stress for the type I nozzle. No measurements of $(uv)'$ in this flow were made. The higher values, in this case, are again indicative of the influence of the shear layers in the early development of the jet.

The spanwise variation $u'(\zeta)$ are shown in Figs. 10a and b. In the flow out of the sharp-edged nozzle, looking at $U(\zeta)$ and $u'(\zeta)$, one notes that the maximum value of U corresponds to a minimum value of u'. The turbulent intensity reaches a maximum near the edge of the jet and $u'(\zeta)$ max is larger than $u'(\eta)$ max. In the case of the jet out of the rectangular nozzle and at ten slot thicknesses, $u'(\zeta)$ is perfectly flat and does not have a maximum near the edge of the jet in the spanwise direction. Such a maximum develops further downstream, but never exceeds the centerline value by more than 10% (as compared to 80% in the case of type I nozzle). As x increases further, the jet tends to axisymmetry, $u'(\zeta)$ becomes similar to the longitudinal turbulence intensity profiles measured in circular jets.

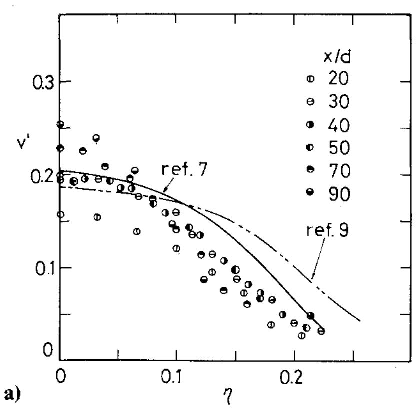


Fig. 8a Transverse turbulent intensity profiles in the *x, y* plane; nozzle type II, *Re* = 12,200.

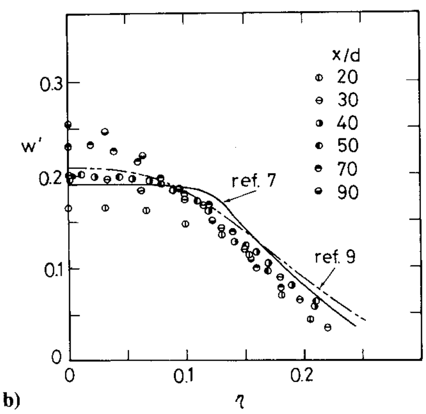


Fig. 8b Lateral turbulent intensity profiles in the *x, y* plane, nozzle type II, *Re* = 12,200.

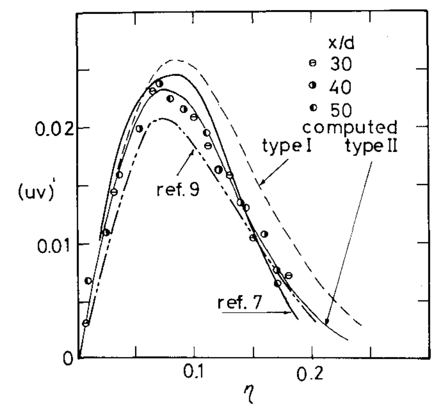
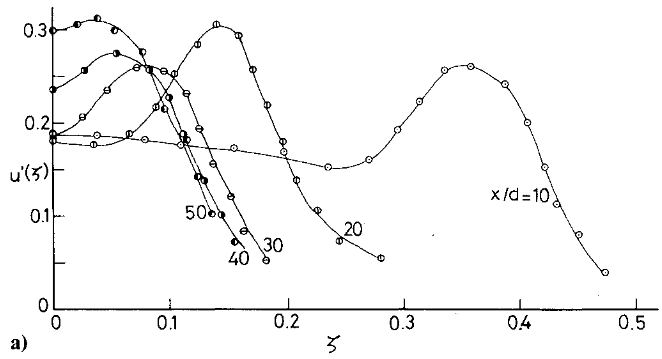


Fig. 9 Turbulent shear stress profiles in the *x, y* plane, *Re* = 12,200.



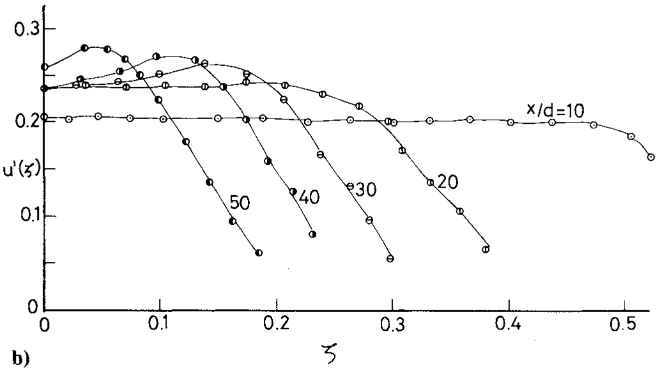


Fig. 10 Longitudinal turbulent intensity profiles in the *x, z* plane: a) nozzle type I, *Re* = 12,200; b) nozzle type II, *Re* = 12,200.

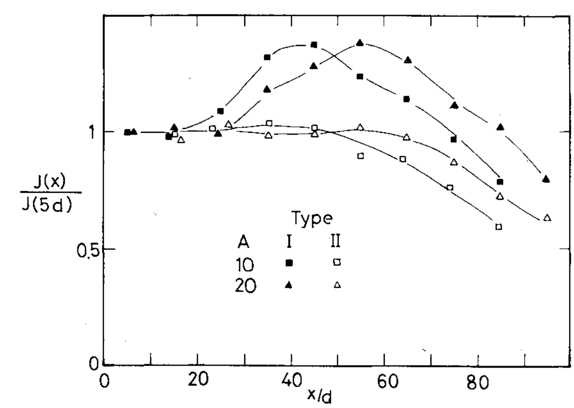


Fig. 11 Momentum flux variation in the *x,y* plane, *Re* = 12,200.

Disussion

Nozzle shape and aspect ratio play a major role in the development of rectangular jets. Measurements made in the smaller jets are qualitatively similar to those reported in Ref. 1. The variation of the mean velocity along the centerline in the so-called characteristic decay region shows some departure from the decay law which characterizes two-dimensional jets. The velocity decay exponent n increases with A and tends to the value $n = 2$, which characterizes the centerline velocity decay of two-dimensional jets. However, n does not vary with A in any systematic manner, as may be noted from Figs. 2a and b. In the case of smaller nozzles, n approaches 2 for $A = 40$, while this value is reached for $A = 10$ in the case of the larger nozzles. Turbulence measurements taken in the smaller jets are consistent with the mean flow and show a similar departure from the behavior of a two-dimensional jet. The turbulent intensity along the centerline does not remain constant for any appreciable length, and stays well below intensity levels usually measured in two-dimensional jets. Here again, the departure from two dimensional behavior is inversely proportional to A.

It is interesting to note that, for a given nozzle type, the three-dimensional effects which cause the departure from two-dimensional behavior are well correlated with the spanwise length scale or A. It has been shown in Ref. 4 that the maximum contraction of the jet in the x,y plane occurs at about $x/d \simeq 3A$. A similar correlation occurs in Fig. 6a, where the minimum turbulent intensity along the centerline occurs at $x/l \simeq A$. For the larger nozzles, Figs. 6b and c show the minimum at $x/l \simeq 2.5\,A$ and $x/l \simeq 4A$, respectively.

The effects of nozzle shape are well illustrated by the detailed measurements of mean and turbulent quantitites for nozzles of types I and II at the same Re and A. The cross section of these jets, which is initially elliptical with the major axis along z, gets strained in the y,z plane as the jet develops further downstream. Eventually, the cross section becomes elliptical, but with the major axis along y. This phenomenon is observed mainly for type I nozzles, and is accompanied by other effects described previously, such as the "saddle-back" shape of $U(\zeta)$, the variation of $u'(x)$ and $u'(\zeta)$. Under such conditions, one cannot speak of self-preservation, even if some features of the flow such as $U(\eta)$ and $u'(\eta)$ prove to be geometrically similar at different stations downstream. In fact, computing the momentum flux at different stations downstream one finds that for nozzle I, with $A=10$; the momentum in the x,y plane increases rapidly for $x>20$. This is shown in Fig. 11, where the momentum in the x,y plane at any station x is normalized with respect to the momentum at $x=5d$. At $x=40d$, the momentum flux in the plane x,y increases by 30% over its initial value. The increase of J is possibly due to some momentum flow from the two sides of the jet as it contracts in the x,y plane.

The flow development in the plane x,y is influenced by two factors—conditions at the jet mouth and three-dimensional effects. The influence of the condition of the jet exit decreases with x, whereas three-dimensional effects increase with x. Two-dimensional self-preserved conditions will be approached if the effects of the initial conditions subside before the appearance of spanwise effects close to the centerline. The development length would be expected to be longer in the case of the jet of nozzle I, as all turbulence is initially confined to the two edges. Furthermore, this jet contracts slowly and has a nonuniform velocity distribution in the x,z plane. As a result, sharp orifice jets, particularly those having a small aspect ratio, are not likely to develop into a self-preserved two-dimensional flow.

A jet out of a long channel develops under more favorable conditions. First, the development length of such a jet would be expected to be shorter and second, the flow is uniform in the x,z plane up to large distances downstream.

It has been suggested that the straining of the jet in the y,z plane may be due to the presence of vortex rings of elliptical shape surrounding the jet.[1,2,6] The "saddle-back" shape of the velocity profiles $U(\zeta)$ is attributed to the velocities induced by these vortices. Sforza et al.[1,2] have observed that the straining of these vortices is very similar to the straining of the rectangular jet. It is difficult to detect the presence of such vortex rings directly, but this view seems to be consistent with the observed results. If these rings existed, they would be expected to be stronger in the case of the sharp orifice jet as the vorticity is initially confined to a very thin layer surrounding the jet. These rings would therefore have more noticeable effects in these jets than in flow out of the channel. The deviation of this jet from two-dimensional self-preserved conditions would be caused by the fact that local invariance cannot be approached until the energy of the vortex rings has completely subsided, i.e., the jet at any section x has become independent of its past history.

Acknowledgments

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